

# TECH TIP # 50



One of a series of dealer contractor technical advisories prepared by HARDI wholesalers as a customer service.

## A Crash Course in Fractions

Here is a quick refresher on working with fractions. This will be A very useful review if you haven't had to work with fractions for awhile.

### Terms

Fractions -- One or more of the equal parts into which a unit may be divided. It is always written in the form of a division problem. As an example, one-eighth indicates one unit divided into eight equal parts and is written  $1/8$ , which would express the fact that it is equal to 1 divided by 8.

Numerator -- The number of equal parts within a fraction. For example, in the fraction  $3/8$ , the number 3 is the numerator of the fraction.

Denominator -- The number of equal parts into which the unit is divided. For example, in the fraction  $3/8$ , the number 8 is the denominator of the fraction.

Proper Fraction -- When the numerator of the fraction is less than the denominator. For example,  $1/2$ ,  $1/4$ ,  $3/4$ , etc., are proper fractions.

Improper Fractions -- The numerator of a fraction is equal to or greater than the denominator. For example,  $8/3$ ,  $4/4$ ,  $20/15$ , etc., are improper fractions.

Mixed Number -- A combination of a whole number and a fraction. For example:  $5 \frac{1}{4}$  and  $3 \frac{1}{2}$ . Mixed numbers are sometimes typed for convenience as  $5-1/4$  and  $3-1/2$ . Also, the mixed number such as  $5 \frac{1}{4}$  means the same as  $5 + \frac{1}{4}$ .

Adding and subtracting fractions with the same denominator -- simply add or subtract numerators and use the common denominators.

Example: add

$$\frac{3}{16} + \frac{9}{16} + \frac{3}{16} = \frac{3+9+3}{16} = \frac{15}{16}$$

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Example: subtract

$$\frac{13}{16} - \frac{7}{16} = \frac{13-7}{16} = \frac{6}{16} = \frac{3}{8} \longleftarrow \text{Lowest terms}$$

Note: In the subtraction example, the fraction 6/16 was reduced to its lowest terms by dividing both the numerator and the denominator by 2 to obtain 3/8. Dividing or multiplying both the top and bottom of a fraction by the same number does not change the value of the fraction.

To add or subtract fractions with different denominators, you must first find a common denominator then add or subtract the resulting numerators.

Examples: add       $\frac{3}{16} + \frac{5}{8} + \frac{5}{32}$

A common denominator is simply any one number that each of the original denominators will divide into evenly -- consider 64 ...

$$\begin{array}{ccc} & \times 4 & \\ \swarrow & & \searrow \\ 3 & & 12 \\ \hline 16 & \longrightarrow & 64 \end{array} = \frac{\quad}{64}$$

Divide 16 into 64 and obtain 4.  
Then multiply 4 times 3 to obtain 12.  
Repeat this procedure for each fraction to obtain all common (but still equivalent) fractions.

$$\frac{5}{8} = \frac{40}{64}$$

$$\frac{5}{32} = \frac{10}{64}$$

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$$\frac{62}{64} = \frac{31}{32}$$

Note: we could have used 32 as the lowest or smallest common denominator and obtained the same result.

When a common denominator is not obvious, one can be determined by using the largest denominator in the group or multiplying two or more denominators together.

To change a whole number to a fraction, simply place it over 1 as a denominator.

Thus, 3 changed to a fraction would be 3/1.

To change a mixed number to a fraction, or more correctly, an improper fraction, multiply the whole number by the denominator and add the numerator. For instance: consider the mixed number 5-7/8:

$$5 \frac{7}{8} = \frac{(5 \times 8) + 7}{8} = \frac{40 + 7}{8} = \frac{47}{8}$$

To convert or simplify an improper fraction to a mixed number, divide the numerator by the denominator and if there is any remainder, it becomes the new numerator of the fraction along side the whole number. For example: consider 43/16

$$\frac{43}{16} = 16 \begin{array}{r} 2 \\ \overline{) 43} \\ \underline{-32} \\ 11 \end{array} = 2 \frac{11}{16}$$

If there is no remainder, the improper fraction simply reduces to a whole number, such as 32/8 reduces to 4.

The most common way to add or subtract mixed numbers is to consider the whole numbers and fractions --- adding or subtracting them accordingly --- then combining the results for the correct answer. Consider the need to add 2-1/4 and 1-3/16.

$$2 \frac{1}{4} = 2 + \frac{1}{4} = 2 + \frac{4}{16}$$

$$1 \frac{3}{16} = 1 + \frac{3}{16} = 1 + \frac{3}{16}$$


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$$3 + \frac{7}{16}$$

16 is common denominator

Subtract 12-1/2 from 27-3/4 ...

$$27 \frac{3}{4} = 27 \frac{3}{4}$$

$$-12 \frac{1}{2} = -12 \frac{2}{4}$$


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$$15 \frac{1}{2}$$

But consider this subtraction problem:  $3\frac{7}{8}$  from  $4\frac{1}{16}$  ...

$$\begin{array}{r} 4\frac{1}{16} \\ -3\frac{7}{8} \\ \hline \end{array}$$

After finding the common denominator for the fractions, note that we cannot subtract  $14/16$  from the smaller value  $1/16$ . To complete this subtraction, we must “borrow” 1 from the whole number 4, reducing the whole number to 3, then add 1 or its fractional equivalent  $16/16$  to  $1/16$  to obtain  $17/16$  which is larger than  $14/16$ .

$$\begin{array}{r} 4\frac{1}{16} = 3\frac{17}{16} \\ -3\frac{14}{16} \\ \hline \phantom{3}\frac{3}{16} \end{array}$$

We can do this because..  $4\frac{1}{16} = 4 + 1/16 = 3 + 1 + 1/16 = 3 + 16/16 + 1/16 = 3 + 17/16$

To multiply fractions, simply multiply the numerators together and place above the line and then multiply the denominators together to form a new denominator. It may then be possible to reduce the resulting fraction to its lowest form. For example: multiply

$$\frac{1}{2} \times \frac{3}{16} = \frac{1 \times 3}{2 \times 16} = \frac{3}{32}$$

Now multiply  $5/9$  times  $3/16$  ....  $\frac{5}{9} \times \frac{3}{16} = \frac{5 \times 3}{9 \times 16} = \frac{15}{144}$

By dividing top and bottom by 3 we can reduce to simplest terms ...

$$\frac{15}{144} = \frac{5}{48}$$

It's also possible to multiply these two numbers by first exercising a *canceling* procedure

$$\frac{5}{9} \times \frac{3}{16} = \frac{5 \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times 16} = \frac{5}{48}$$

Note in the above approach, canceling was simply noting that 3 in the numerator would divide evenly into the 9 in the denominator to reduce the size of each number without changing the relationships. If 16 in the denominator had been 15, then we could have also cancelled 5 into 15 .....

$$\frac{5}{9} \times \frac{3}{15} = \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{3}{\cancel{15}}} = \frac{1}{9}$$

To multiply mixed numbers, change the mixed numbers to fractions and then proceed as for fractions.

Example:  $3\text{-}1/2$  times  $4\text{-}3/4$ .

Change to improper fractions...  $3\frac{1}{2} = \frac{7}{2}$

$$4\frac{3}{4} = \frac{19}{4}$$

Then ....  $\frac{7}{2} \times \frac{19}{4} = \frac{7 \times 19}{2 \times 4} = \frac{133}{8}$

Convert to a mixed number...  $\frac{133}{8} = 16\frac{5}{8}$

To divide a fraction by a fraction, multiply the first fraction by the reciprocal (reverse) of the other.

For example: divide  $11/9$  by  $2/5$  ...

reversed  $\frac{11}{9} \times \frac{5}{2} = \frac{55}{18} = 3\frac{1}{18}$

To divide mixed numbers, change the mixed numbers to improper fractions and proceed as detailed above. Example:  $2\text{-}3/4$  divided by  $1\text{-}5/8$

Change to improper fractions.....

$$2\frac{3}{4} = \frac{11}{4}$$

$$1\frac{5}{8} = \frac{13}{8}$$

$$\frac{11}{4} \text{ divided by } \frac{13}{8} = \frac{11}{4} \div \frac{13}{8} = \frac{11}{4} \times \frac{8}{13} = \frac{22}{13}$$

← reversed

↑ indicates division

$$\frac{22}{13} = 1\frac{9}{13}$$

### Practice Problems

#### Change to improper fractions

1.  $1\frac{1}{2}$       2.  $5\frac{2}{3}$       3.  $10\frac{25}{32}$       4.  $100\frac{3}{4}$       5.  $987\frac{57}{64}$

#### Reduce to lowest terms

6.  $\frac{32}{64}$       7.  $\frac{27}{30}$       8.  $\frac{64}{32}$       9.  $\frac{100}{7}$       10.  $\frac{1000}{64}$

#### Add

11.  $\frac{1}{16} + \frac{3}{16} + \frac{5}{16} + \frac{7}{16} =$       12.  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} =$
13.  $\frac{2}{7} + \frac{3}{9} =$       14.  $1\frac{3}{8} + \frac{13}{15} + 5\frac{25}{32} =$
15.  $\frac{3}{4} + \frac{5}{16} + 12\frac{1}{2} + \frac{5}{8} =$

#### Subtract

16.  $\frac{3}{4} - \frac{1}{2} =$       17.  $\frac{5}{16} - \frac{9}{32} =$       18.  $1\frac{17}{32} - \frac{11}{27} =$
19.  $7\frac{3}{16} - \frac{7}{16} =$       20.  $175\frac{2}{3} - 15\frac{27}{32} =$

### Multiply

21.  $\frac{1}{2} \times \frac{1}{2} =$

22.  $\frac{3}{16} \times 4\frac{2}{3} =$

23.  $3\frac{3}{4} \times 4\frac{4}{5} =$

24.  $\frac{25}{32} \times 2\frac{7}{10} \times 12\frac{2}{3} =$

25.  $5\frac{33}{200} \times 2\frac{1}{2} \times \frac{20}{1033} =$

### Divide

26.  $\frac{1}{2} \div \frac{1}{2} =$

27.  $4 \div \frac{4}{5} =$

28.  $\frac{3}{16} \div 6 =$

29.  $11\frac{21}{33} \div 10\frac{12}{15} =$

30.  $100 \div \frac{1}{10} =$

### Answers

1.  $\frac{3}{2}$

2.  $\frac{17}{3}$

3.  $\frac{345}{32}$

4.  $\frac{403}{4}$

5.  $\frac{63225}{64}$

6.  $\frac{1}{2}$

7.  $\frac{9}{10}$

8. 2

9.  $14\frac{2}{7}$

10.  $\frac{125}{8} = 15\frac{5}{8}$

11.  $\frac{16}{16} = 1$

12.  $\frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$

13.  $\frac{18}{63} + \frac{21}{63} = \frac{39}{63} = \frac{13}{21}$

14.  $1\frac{180}{480} + \frac{416}{480} + 5\frac{375}{480} = 6\frac{971}{480} = 8\frac{11}{480}$

15.  $\frac{12}{16} + \frac{5}{16} + 12\frac{8}{16} + \frac{10}{16} = 12\frac{35}{16} = 14\frac{3}{16}$

16.  $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

17.  $\frac{10}{32} - \frac{9}{32} = \frac{1}{32}$

18.  $1\frac{459}{864} - \frac{352}{864} = 1\frac{107}{864}$

19.  $6\frac{19}{16} - \frac{7}{16} = 6\frac{12}{16} = 6\frac{3}{4}$

20.  $175\frac{64}{96} = 174\frac{160}{96} - 15\frac{81}{96} = 159\frac{79}{96}$

$$21. \frac{1}{4}$$

$$22. \frac{\cancel{3}}{16} \times \frac{7}{\cancel{14}} = \frac{7}{8}$$

$$23. \frac{\cancel{15}}{4} \times \frac{6}{\cancel{5}} = \frac{18}{1} = 18$$

$$24. \frac{\cancel{25}}{16} \times \frac{9}{\cancel{10}} \times \frac{19}{\cancel{38}} = \frac{855}{32} = 26 \frac{23}{32}$$

$$25. \frac{\cancel{1033}}{200} \times \frac{1}{\cancel{5}} \times \frac{1}{\cancel{20}} = \frac{1}{4}$$

$$26. \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{1} = \frac{1}{1} = 1$$

$$27. \frac{4}{1} \times \frac{5}{4} = \frac{5}{1} = 5$$

$$28. \frac{\cancel{3}}{16} \times \frac{1}{\cancel{6}} = \frac{1}{32}$$

$$29. \frac{\cancel{384}}{11} \times \frac{5}{\cancel{15}} = \frac{960}{891} = 1 \frac{69}{891}$$

$$30. \frac{100}{1} \times \frac{10}{1} = 1000$$